Assignment 2

Solve the following exercises. You must submit one and only one .doc/.docx file containing text describing your findings and what you did to reach them, as well as the R code you used. Zip/RAR files are not allowed.

1. A container holds 10 white balls and 5 + K black balls, where K is the last digit of your student ID. We then draw a ball at random. If the ball we choose is white or black, we return it along with another ball of the same color and one more red ball. (For example, if we draw a white ball, we return it to the container with another white ball and a red ball, so if we had 10 white and 10 black, we would now have 11 white, 10 black, and one red). If we choose a red ball at any point, we simply return it without adding another ball. (a) If we continue to choose one ball at a time and repeat this 100 times, how many white and black balls will we have at the end of the experiment? Use Monte Carlo and write an R function to implement the above experiment. (b) Repeat the experiment 1000 times on the computer (i.e., perform 100 draws, and after completing each repetition, record what you have after the 100 draws). So now you will have 1000 values for the percentage of white, black, and red balls. Find descriptive measures to describe your result as best as possible. Comment. (c) Now forget the red balls. Each time we draw a ball, we simply return it along with a ball of the same color. Starting again with 10 white and 5 + K black balls, what is the probability that if you repeat the process 1000 times (so you start with 15+K balls and will end up with 1015+K balls), you will at some point achieve a sequence of 10 white balls?

2. Consider the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Someone randomly chooses 5 of these without the possibility of choosing any digit two or more times. What is the probability that they choose them such that the digits are in increasing order? (a) Answer by creating all possible 5-tuples and counting the favorable cases. (b) Using Monte Carlo.

3. Consider the following game: Heads (H) or Tails (T). A player chooses a sequence of size 3 that they believe will occur. For example, they choose HHT, meaning that a sequence of three flips will be Heads, Heads, Tails. The opponent chooses another sequence, e.g., THT. Then, they flip the coin until one of the two sequences appears. For the previous example, suppose the sequence is THTHH, so the sequence chosen by player B appeared first, and therefore B won. (a) Write a function that takes the two sequences as input and flips the coin until a winner is found. (b) If player A chooses the sequence HHT, what is the probability that they will win if player B chooses HHH? Explain with numbers how you arrived at the conclusion. (c) Given that with 3 coin flips, all possible triplets are 8, so player B has 7 choices. Which of the 7 is the one that gives them the highest probability of winning?

4. Consider the integral:

$$I = \int_{-1}^{2} \exp(-x^2/2)dx$$

(a) Calculate it using Monte Carlo.

(b) Calculate it using the geometric interpretation.

(c) Using Monte Carlo but in a different way than in (a).

(d) Use the trapezoidal rule to calculate the integral.

Important: In each case, you should have an accuracy of 2 decimal places, meaning that every time you run the code, you will get the same first 2 decimal digits. How does this affect the number of your repetitions?

5.Consider the following game with three dice. The dice are custom-made: • Die A has sides 2, 2, 4, 4, 9, 9. • Die B has sides 1, 1, 6, 6, 8, 8. • Die C has sides 3, 3, 5, 5, 7, 7. Suppose your opponent chooses die A. You choose a die as well, let's say die C. You both roll your dice, and the one with the higher number wins. The game is that simple. (a) Who is more likely to win? Note: You can see this using both Monte Carlo and by doing calculations. (b) Suppose your opponent had chosen die B. Would you still choose die C to have a higher probability of winning against them?

6.Two teams have reached the final of the playoffs. They will play based on a "best of 7" series, meaning the team that first scores 4 wins will win. If the probability of team A winning is 0.65, and assuming that this probability remains constant in each game: (a) What is the probability that team A will win the playoffs? (b) What is the probability that the final will be decided in 5 games? (c) Let's make it even more realistic. The teams initially play 2 games at team A's home, then 2 games at team B's home, and then alternate between the two home venues. If the probability of team A winning at home is 0.70 and winning away is 0.60, what is now the probability that team A will win the playoffs?

7. A four-digit number x is represented as **ABCD**, where **A, B, C, D** are its four digits. For example, the number 4235 has $A = 4, B = 2, C = 3, D = 5$. The number cannot begin with zero. Find which four-digit numbers satisfy the condition:

$$A^B \cdot C^D = x$$

Example: The number 4235 does **not** satisfy the condition because

$$4^2 \cdot 3^5 = 16 \cdot 243 = 3888 \neq 4235$$

8 .A treasure chest contains 4 copper coins, 4 silver coins, and 5 gold coins. When Midas randomly touches any coin of any color, it magically disappears and is replaced by two new coins that have the other two colors. For example, if Midas touches a silver coin, it transforms into one copper coin and one gold coin. (a) After two consecutive random touches by Midas, what is the probability that the gold coins are still more numerous than either of the other two colors? (b) If the number of touches increases, will the probability increase or decrease? Try with different values to answer. (c) Make a graph of the probabilities you found. What do you observe? (d) If we start with 5 copper coins, 5 silver coins, and 6 gold coins, do you think anything will change? Note: This problem has several applications in economics as it describes a case of competition between different products.